



## ASHTON HAYES PRIMARY SCHOOL

### Mathematical Fluency, Reasoning and Problem Solving

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Following professional dialogue amongst Ashton Hayes Primary School teaching staff, our shared understanding of what mathematical fluency, reasoning and problem solving means is detailed below.

#### Mathematical Fluency

The first aim of the new mathematics national curriculum in England (DfE, 2013) is that all pupils will: *'become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.'*

#### At Ashton Hayes Primary School:

- ⇒ We think that the key to mathematical fluency is in making connections, and making them at the right time in a child's learning.
- ⇒ We believe fluency demands more of children than memorising a single procedure. The children need to understand why they are doing what they are doing and know when it is appropriate to use different methods.

#### Thoughts about mathematical fluency based on Russell (2000):

Mathematical Fluency consists of three elements



#### Efficiency

- This implies children don't get bogged down in too many steps. An efficient strategy is one that the child can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problems.



#### Accuracy

- This depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.



#### Flexibility

- This requires the knowledge of more than one approach to solving a particular kind of problem (e.g. 2-digit multiplication). Children need to be flexible in order to choose an appropriate strategy for the numbers involved, and also be able to use one method to solve a problem and another to check the results.

## Why do children need to be mathematically fluent?

*To the person without number sense, arithmetic is a bewildering territory in which any deviation from the known path may rapidly lead to being totally lost.*

**Dowker (1992)**

*The phrase 'number sense' is often used to mean conceptual fluency – understanding place value and the relationships between the operations. Children need to be both procedurally and conceptually fluent – they need to know both how and why. Children who engage in a lot of practice without understanding what they are doing often forget, or remember incorrectly, those procedures. Further, there is growing evidence that once pupils have memorised and practice procedures without understanding, they have difficulty learning later to bring meaning to their work.*

**Hiebert (1999)**

## How can we support children in becoming fluent?



Manipulatives/Apparatus: e.g. Numicon, dienes, Base 10, cubes, bead-strings.



Quality talk about work. Not just sharing how they did a particular calculation but describing why and how it worked and how their method is the same or different to those of others.



Consolidation in meaningful contexts: By offering children practice in context we help them to make links between the types of situations that a particular strategy might suit. Russell calls this mathematical memory, which is different from just memorising.

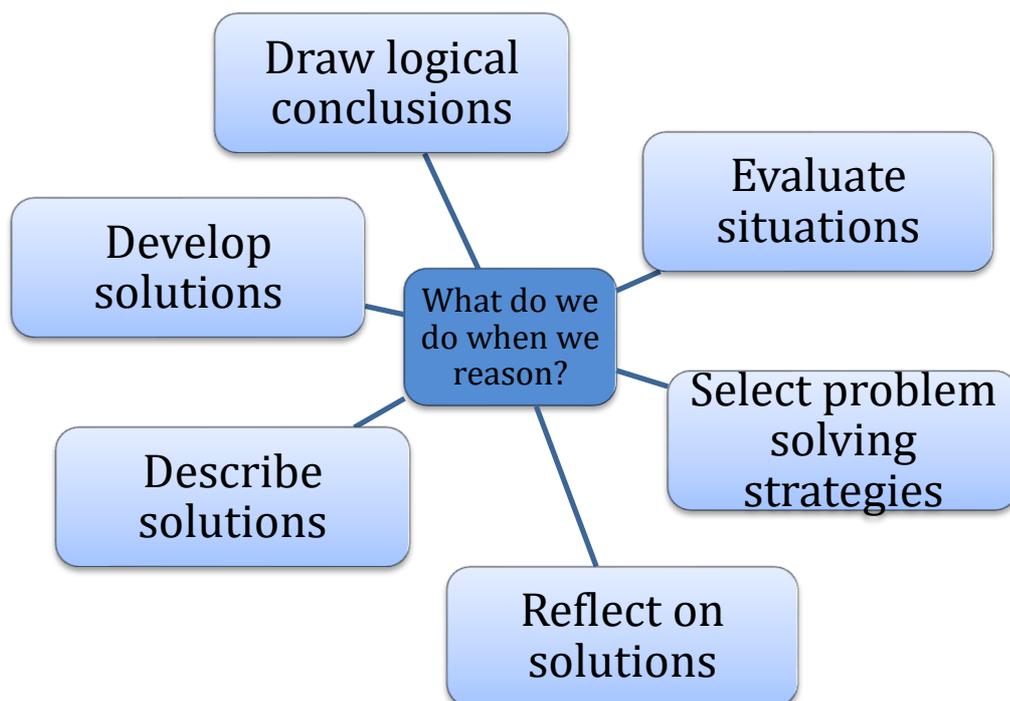
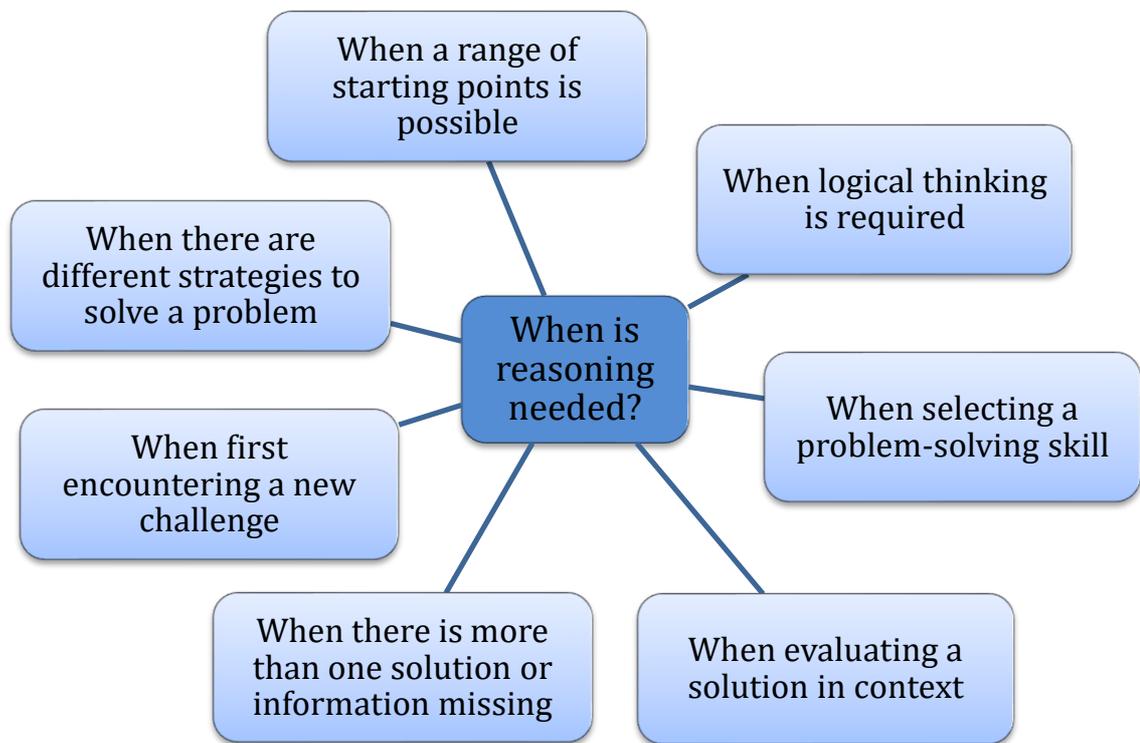
### Mathematical Reasoning

The second aim of the new mathematics national curriculum in England (DfE, 2013) is that all pupils will:

*'reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.'*

### At Ashton Hayes Primary School:

- ⇒ We think that reasoning is fundamental to knowing and doing maths.
- ⇒ We believe reasoning enables children to make use of all their other mathematical skills.
- ⇒ We find it helpful to explain reasoning as the 'glue' that helps maths make sense.



### A possible progression in reasoning

A five-step progression in **reasoning**: a spectrum that shows us whether children are moving on in their reasoning from novice to expert. We believe that children are unlikely to move fluidly from one step to the other, rather flow up and down the spectrum settling on a particular step that best describes their reasoning skills at any one time.

*Step one:* Describing: simply tells what they did.

*Step two:* Explaining: offers some reasons for what they did. These may or may not be correct. The argument may yet not hang together coherently. This is the beginning of inductive reasoning.

*Step three: Convincing:* confident that their chain of reasoning is right and may use words such as, 'I reckon' or 'without doubt'. The underlying mathematical argument may or may not be accurate yet is likely to have more coherence and completeness than the explaining stage. This is called inductive reasoning.

*Step four: Justifying:* a correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'therefore', 'and so', 'that leads to' ...

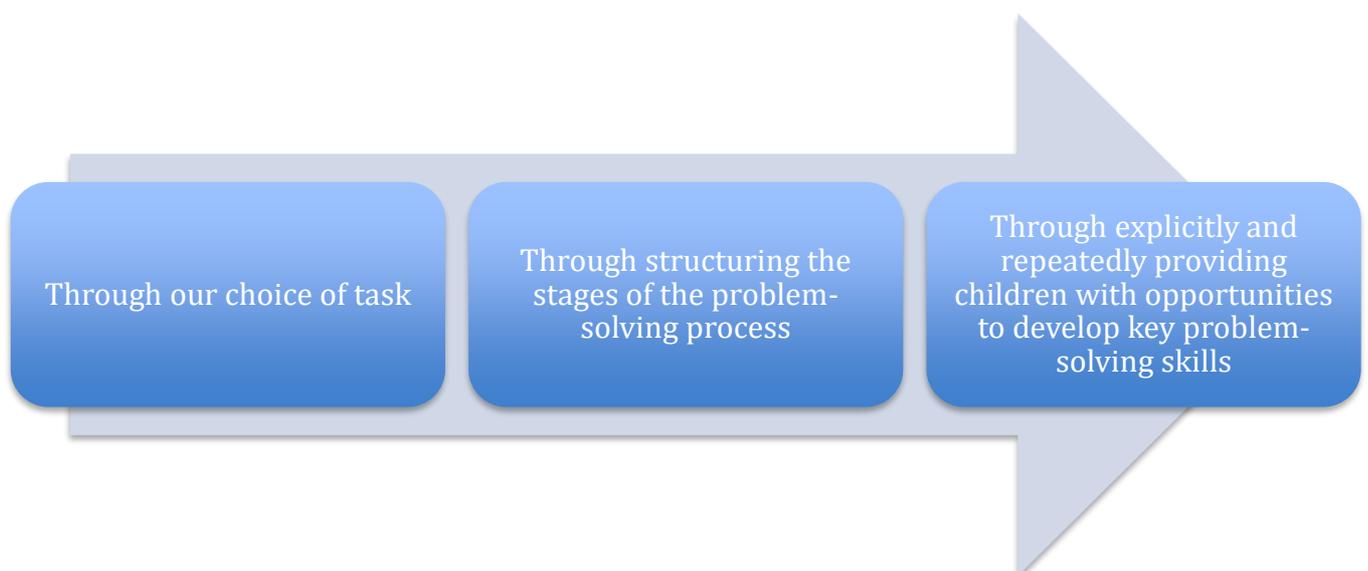
*Step five: Proving:* a watertight argument that is mathematically sound, often based on generalisations and underlying structure. This is also called deductive reasoning.

### **Mathematical Problem Solving**

The third aim of the new mathematics national curriculum in England (DfE, 2013) is that all pupils: *'can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.'*

### **At Ashton Hayes Primary School:**

- ⇒ We think that becoming a competent and confident problem solver is central to the mathematical development of all learners.
- ⇒
- ⇒ We believe that we should strive to put problem solving at the heart of our maths teaching – it shouldn't be an optional extra for Friday afternoons or a special activity to be done when children have finished everything else.
- ⇒
- ⇒ We think that becoming confident and competent as a problem solver is a complex process that requires a range of skills and experiences.
- ⇒ We can support this process in three principal ways



### **The stages of the problem-solving process**

The problem-solving process can usually be thought of as having four stages:

- Stage 1: Getting started
- Stage 2: Working on the problem

- Stage 3: Digging deeper
- Stage 4: Concluding

We can helpfully spend time with children concentrating on one of these stages explicitly, in turn, as they learn to become confident problem solvers.

**Stage 1: Getting started** will mean offering them strategies to help them engage with the problem. These could be prompts such as:

- Tell me/a partner what you think the problem is about.
- What would help you understand the problem?
- You might like to draw a diagram, act it out or represent it with a model.
- What other problems have you seen that are 'a bit like' this one?
- What mathematical skills have you got that could be helpful here?
- Try making a simpler case to get an idea of how the problem works.

**Stage 2: Working on the problem** will usually involve using one or several problem-solving skills such as:

- Trial and improvement
- Working systematically (and remember there will be more than one way of doing this: not just the one that is obvious to you!)
- Pattern spotting
- Working backwards
- Reasoning logically
- Visualising
- Conjecturing.

These problem-solving skills are in a random order, although the first two, trial and improvement and working systematically, are key skills that will support children to become competent as problem solvers. The children will benefit from becoming proficient in each of these skills and working on one of them as a key focus in a lesson or series of lessons could be a useful strategy.

**Stage 3: Digging deeper** usually happens when the problem has been explored and then it is possible to look for generalisations and proof.

**Stage 4: Concluding** is the part of the problem-solving process where we support the children to learn to explain their findings both verbally and in writing. They may need some support to explain succinctly, use words such as 'because' and to use the appropriate mathematical vocabulary correctly. This all takes time, attention and practice. Written recording could be in the form of a photograph, diagram or written explanation. Children will need support to develop their proficiency with written recording.

Also in the concluding part of the problem-solving adventure children will need to be supported to compare different strategies that were used to solve the problem in order to consider the efficiency of the method and the elegance of the solution. This will enable them to see how they might refine their own methods or adopt a different one next time they encounter a similar problem.

## Mathematical Problem Solving in the Early Years:

In order to support children in the early development of mathematics into becoming confident and competent problem solvers, there are arguably two crucial choices for practitioners:

**Choice of 'task' (rich context): think resources and skilful questioning**

A rich context could be thought of as a carefully chosen, inviting set of resources that offer lots of freedom to play, explore, question and try out ideas. The resources themselves may not be regarded as inherently mathematical.

**Choice of approach: think follow, support, question, and stimulate thinking**

The next step is the ability of the adults in the setting to recognise the mathematical potential in play activities and draw this out through skilful questioning. It is central to this approach that the adult follows and stimulates the child's thinking. The adult needs to resist any temptation to 'lead', 'tell' or 'impose ideas' on what the child is doing. Their challenge is to seek to understand what the child is doing and thinking, and support them to develop their ideas further. Throughout this process the adult is seeking to develop the children's key problem-solving skills in a context of the child's choosing.

Prompts to further support the children in their mathematical thinking and problem solving could be grouped into the following four categories:

**Describing** – prompts which encourage children to talk about their mathematics, which helps organise their thoughts and helps familiarise them with mathematical language

**Recording** – prompts which encourage children to think about how they could keep a record of what they have done, whether to help them explore the mathematics in that moment or to refer back to at another time themselves or to communicate to someone else what they have done

**Reasoning** – prompts which encourage children to connect ideas together, perhaps making logical arguments and to go beyond describing what they have done to explaining why

**Opening out** – prompts that encourage children to explore the context further and possibly more deeply.